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Dynamic Aperture Studies for SPEAR 3¹

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Abstract. The Stanford Synchrotron Radiation Laboratory is investigating an accelerator upgrade project that would replace the present 130 nm-rad FODO lattice with an 18 nm-rad double bend achromat (DBA) lattice: SPEAR 3. The low emittance design yields a high brightness beam, but the stronger focusing in the DBA lattice increases chromaticity and beam sensitivity to machine errors. To ensure efficient injection and long Touschek lifetime, an optimization of the design lattice and dynamic aperture has been performed. In this paper, we review the methods used to maximize the SPEAR 3 dynamic aperture including necessary optics modifications, choice of tune and phase advance, optimization of sextupole and coupling correction, and modeling effects of machine errors, wigglers and lattice periodicity.

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INTRODUCTION

SPEAR 3 is the 3 GeV upgrade project under study at SSRL [1]. It aims at replacing the current 130 nm·rad FODO lattice with an 18 nm·rad low emittance double bend achromat (DBA) lattice. To minimize the cost of the project and to use the existing synchrotron light beam lines, the new design [2,3] closely follows the racetrack configuration of the SPEAR tunnel, with the magnet positions fit to the 18 magnet girders shown in Fig. 1. As in the current design, the SPEAR 3 lattice has two-fold symmetry and periodicity with two identical arcs and two long straight sections. Each arc in the new lattice has 7 identical symmetric cells, and each straight section consists of two mirror symmetric matching cells.

The lattice functions of one quarter of the ring are shown in Fig. 2. The two bends and a quadrupole in the middle of each DBA cell compensate the dispersion, while the two quadrupole doublets at each end control the tune and cell β functions. Since the new lattice cells have to fit to the existing 11.7 m cell length, it results in a compact DBA design, and hence increases the focusing. Similar to other light source lattices, it has been found advantageous to add vertical focusing to the bends [5–8] to relax the optics and reduce the strength of cell sextupoles by increasing the difference in their β functions [9]. Each matching cell has an extra quadrupole for a better optics matching, two 3/4 bends, with magnet positions adjusted to maintain the ring circumference [4] (the current 358.53 MHz RF system will be used), and quadrupole strengths adjusted to optimize β functions and phase advance.

Though the DBA design has an advantage of a high brightness beam, its effect on the beam dynamics has to be verified. First, the lower emittance results in a higher

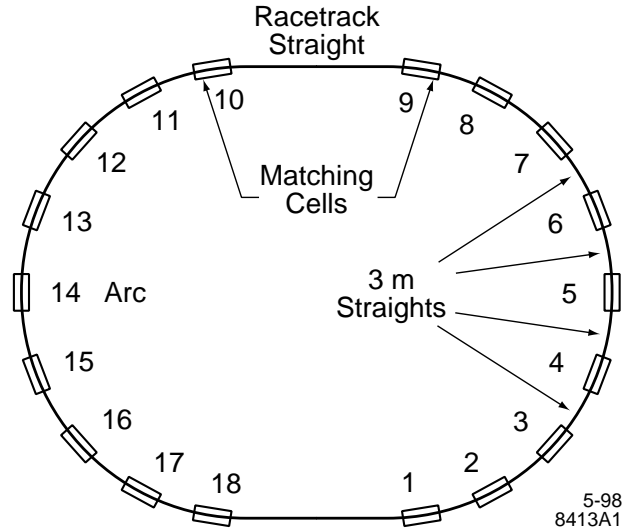


FIGURE 1. Schematic of SPEAR tunnel girders.

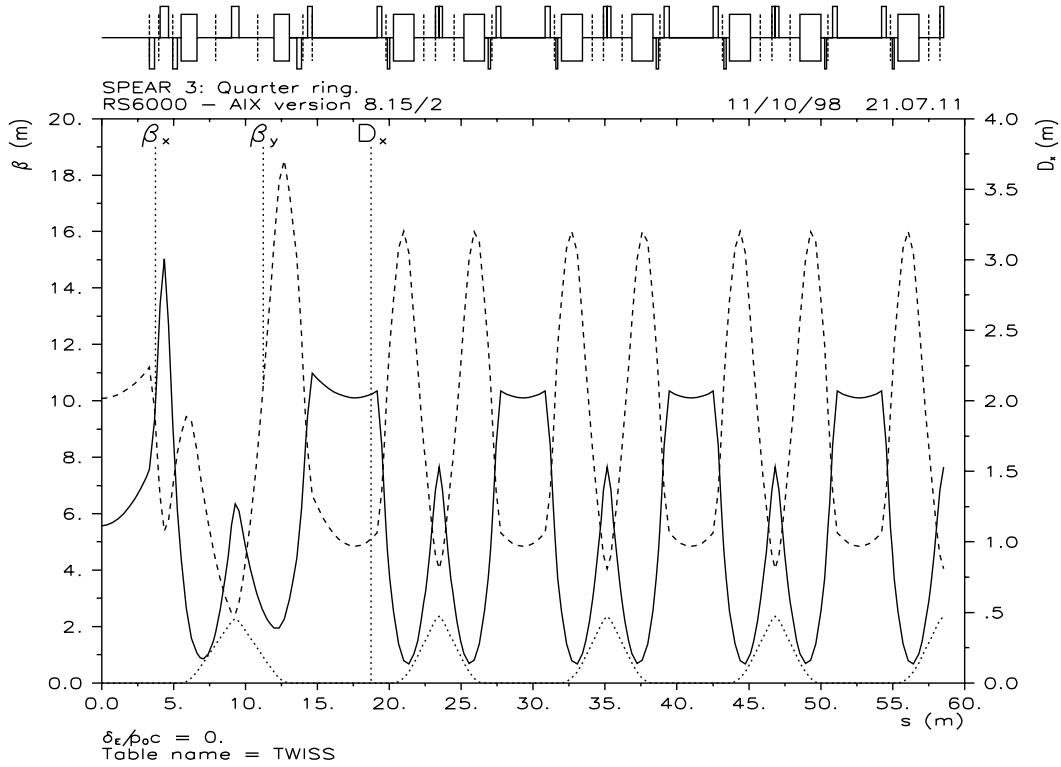


FIGURE 2. Optics of One Quadrant of SPEAR 3 (from the center of the long straight section to the middle of the arc).

particle density which increases probability of particle collisions inside the bunches and makes the Touschek effect the limiting factor for the beam lifetime. Secondly, the stronger focusing in the DBA lattice increases beam sensitivity to magnetic and chromatic errors and generates larger chromaticity. The latter requires strong sextupole correctors which increase the amplitude dependent and non-linear chromatic aberrations. These effects tend to reduce the dynamic aperture, and if the aperture is not sufficient for momentum errors up to $\delta = 3\%$ it will further reduce the Touschek beam lifetime. Adequate dynamic aperture is also important in order to minimize losses during horizontal oscillations of the injected beam.

Consequently, the low emittance design has to be optimized to achieve the maximum dynamic aperture. It is especially important to maximize the horizontal size of dynamic aperture to minimize the Touschek effect and allow large injection oscillations in SPEAR 3. The improvement of dynamic aperture starts from optimization of linear optics and correction systems. In the following sections we review the modifications made to the initial lattice design and present tracking studies including effects of magnetic and chromatic errors, perturbation due to wigglers, and the effect of lattice periodicity.

All of the tracking study has been done using the LEGO code [10] which employs

element-by-element tracking based on symplectic integration techniques [11]. In a few cases we tested LEGO results against other available tracking codes and found good agreement. In our study we calculated dynamic aperture at injection point located at the symmetry point between arc cells where $\beta_x = 10.1$ m and $\beta_y = 4.8$ m. The other typical parameters were: number of tracking turns $N = 1024$, linear chromaticity corrected to zero, and synchrotron oscillations included.

ERROR FREE DYNAMIC APERTURE

Typically, the error free dynamic aperture serves as an upper limit for the aperture with machine errors or with insertion devices (ID). It is therefore important to maximize first the dynamic aperture for the ideal lattice without any magnetic or misalignment errors. Maximizing the error free dynamic aperture necessarily involves optimization of linear optics and the chromaticity correction system, minimization of chromatic and high order effects, and optimizing the betatron tune.

Cell Optics

The DBA cell optics was made to fit the existing 11.7 m cell length with the magnet positions constrained to provide ~ 3 m space for the insertion devices and with bend positions kept to fit to the current synchrotron light beam lines. This results in a compact DBA design which leads to stronger focusing and, hence, increased beam sensitivity to machine errors. Though the achromat lattice eliminates dispersion in the insertion devices and at injection, it limits the available positions for chromatic sextupoles to rather short dispersive regions between the bends and the middle quadrupole QFC. The close proximity of the SF and SD sextupoles reduces their effectiveness and requires larger strengths.

To relax the cell optics it has been found advantageous to add vertical focusing to the cell bends. This results in a better separation of horizontal and vertical focusing and reduces the quadrupole strength in the doublets. Most importantly, due to increased β_y at SD sextupoles it provides better optical separation between the SF and SD and reduces their strength. To further reduce the strengths of the sextupoles and QFC quadrupole, the bends were placed as far from the cell center as it is possible within existing geometric constraints.

Working Tune and Phase Advance

The choice for the phase advance in the arc cells was to be near $\mu_x \approx 0.75 \times 2\pi$ and $\mu_y \approx 0.25 \times 2\pi$. This provides favorable conditions for local cancellation of: 1) geometric aberrations from arc sextupoles located $-I$ apart, and 2) first order chromatic beta waves from sextupoles and quadrupoles located 90° apart, as well as systematic quadrupole errors. It is worth to note that the high horizontal phase advance is due to the achromat design which requires $\mu_x = \pi$ between the bends.

Initially, the matching cells were designed to provide I -transformation between the two arcs in both planes [12]. This would give the advantage of having effectively a 14-period lattice since the matching cells would be virtually invisible in the first order to the on-momentum particles. Generally, the high periodicity optics provides better cancellation of systematic errors.

With the above choices the total tune would be near $\nu_x \approx 14.5$ and $\nu_y \approx 5.5$. To move the working tune away from the half integer resonance, the phase advance in arc and/or matching cells has to be adjusted. To minimize the resistive wall impedance effects [13] it is favorable to move the tune into the lower quarter on the tune plane ($\nu < 1/2$). Tracking studies showed that relaxing the phase advance through the matching cells improves the SPEAR 3 off-momentum dynamic aperture since it reduces the chromaticity and the strengths of matching quads. On the other hand, relaxing the arc cells would increase the arc sextupole strengths because of unfavorable change of the cell β functions at the sextupoles.

The optimum phase conditions have been identified by performing a horizontal dynamic aperture scan across the matching cell phase advance μ_x and μ_y . The on-momentum and off-momentum dynamic aperture were maximized at about $\mu_x = 0.78 \times 2\pi$ and $\mu_y = 0.42 \times 2\pi$ per matching cell.

To minimize the effect of strong low order betatron resonances the location of the working tune on the tune plane has been chosen slightly below .25, away from the 3rd and 4th order resonance lines. The final choice ($\nu_x = 14.19$, $\nu_y = 5.23$) was based on favorable horizontal injection conditions and the results of dynamic aperture tune scan. The two dimensional diagram of horizontal dynamic aperture (in number of σ_x) versus x and y tune is shown in Table 1, where $\sigma_x \approx 0.45$ mm is the horizontal rms beam size at injection point. During the aperture scan the tune was varied by changing arc phase advance, and the lattice was kept matched at all tunes. The above scan also included a set of random machine errors which will be described in the following sections. With the chosen tune, the phase advance per arc cell is $\mu_x = 0.7907 \times 2\pi$ and $\mu_y = 0.2536 \times 2\pi$. The location of the working tune on the tune plane along with betatron resonance lines up to 4th order is shown in Fig. 3.

Chromatic Correction

Efficient chromatic correction is essential for a large off-momentum dynamic aperture and for a long Touschek lifetime. Since the sextupoles also give rise to non-linear geometric aberrations it is important to minimize these effects by using compensation techniques and reducing the sextupole strengths.

As mentioned previously, the choice for phase advance in the arc cells provides conditions for local compensation of sextupole geometric aberrations and chromatic beta waves from arc quadrupoles and sextupoles. This scheme would work optimally if the number of arc cells was $4 \times \text{integer}$. With 7 arc cells in the design, however, this correction is not complete, and the geometric constraints do not allow

TABLE 1. Horizontal Dynamic Aperture (in number of $\sigma_x = 0.45$ mm) versus ν_x , ν_y . The Tracking Included a Set of Random Machine Errors and 1% Momentum Error.

$\nu_x \rightarrow$ $\downarrow \nu_y$.16	.17	.18	.19	.20	.21	.22	.23	.24	.25	.26	.27	.28	.29	.30
.30	49	49	49	49	51	46	47	47	46	44	41	38	36	49	48
.29	51	46	52	51	48	46	47	47	47	44	41	39	46	49	48
.28	49	49	50	51	49	48	51	49	46	44	42	40	36	48	48
.27	50	49	49	51	50	48	46	49	46	42	40	40	37	38	37
.26	49	46	47	51	49	46	44	47	43	42	42	41	41	39	41
.25	47	47	49	51	49	48	44	43	46	45	42	42	37	41	49
.24	47	47	46	50	47	49	45	44	48	46	43	41	45	43	50
.23	48	49	46	49	48	48	46	50	48	45	44	49	47	42	42
.22	48	48	45	44	49	48	50	48	48	45	46	42	42	43	43
.21	49	45	46	46	48	50	50	51	48	46	44	44	46	46	44
.20	48	48	44	46	41	47	44	50	48	45	46	42	47	48	45
.19	48	46	45	50	48	45	47	49	46	47	47	48	47	47	45
.18	47	45	47	51	49	47	47	42	46	47	43	49	46	43	40
.17	53	49	48	49	50	48	47	45	45	43	46	40	45	44	42
.16	51	49	48	50	49	50	47	47	44	42	41	36	42	41	40

for 8 identical arc cells.

The chromaticity correction using only two sextupole families in the arc cells does not provide adequate dynamic aperture for off-momentum particles. Since the two family sextupoles can only compensate the linear chromaticity, the off-momentum aperture is mostly affected by the non-linear chromatic effects. The reason for the large high order chromaticity is due to the matching cells which contribute about 20% to the total chromaticity and break periodicity of the 14 arc cells. Two additional families of sextupoles (SFI, SDI) placed in the matching cells, similar to the arc cells, help to reduce the non-linear terms and significantly improve the off-momentum dynamic aperture. Table 2 compares HARMON [14] calculations of the high order chromaticity for the lattice with and without SFI, SDI sextupoles.

TABLE 2. 2nd and 3rd Order Chromaticity for SPEAR 3 with and w/o Matching Cell Sextupoles.

2nd and 3rd Order Chromaticity	$\frac{d\nu_x}{d\delta^2}$	$\frac{d\nu_y}{d\delta^2}$	$\frac{d\nu_x}{d\delta^3}$	$\frac{d\nu_y}{d\delta^3}$
w/o SFI/SDI	-117	-52	-674	-301
with SFI/SDI	-48	-12	-228	74

The matching cell sextupoles also generate geometric aberrations and therefore have to be kept relatively weak in order to preserve the on-momentum aperture. The optimum strengths of the matching cell sextupoles have been evaluated by

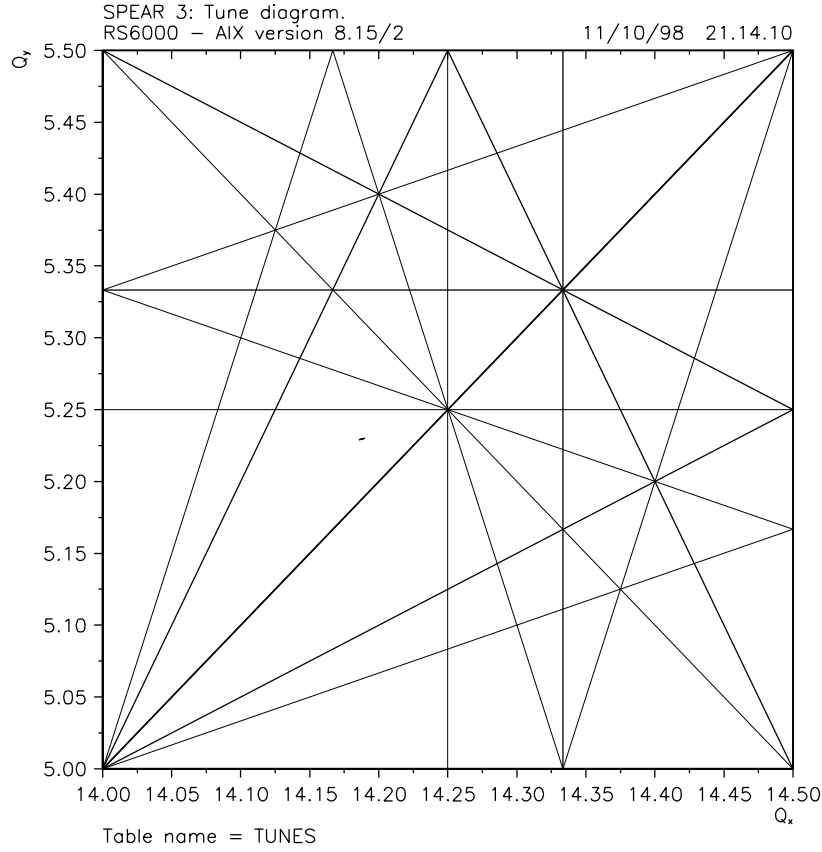


FIGURE 3. SPEAR 3 Working Tune on the Tune Plane.

performing a horizontal aperture scan versus SFI, SDI strengths while using the arc sextupoles to keep the ring chromaticity constant. Other chromatic correction schemes which use more sextupole families in the arcs did not result in a better aperture.

As it was mentioned earlier, the close proximity of the SF, SD sextupoles in the short dispersive region between bends and QFC quadrupole reduces the effectiveness of the sextupoles and increases their strength. To increase the optical separation of the sextupoles two other options were studied. In one option, the SD sextupole was moved away from the SF by combining with part of the adjacent bend. This increased the β_y function at the SD, but the dispersion inside the bend was rather low. The tracking study showed that the dynamic aperture reduces in this option. In the second study, the SF sextupole was combined with the QFC quadrupole. Due to higher dispersion and β_x at the QFC, this led to weaker SF sextupoles and potentially reduced high order sextupole effects. The dynamic aperture, however, did not improve using this option. Consequently, in the current design the sextupoles are kept separate from the bends and QFC quadrupole.

The resultant error free dynamic aperture for on-momentum and $\delta = 3\%$ particles

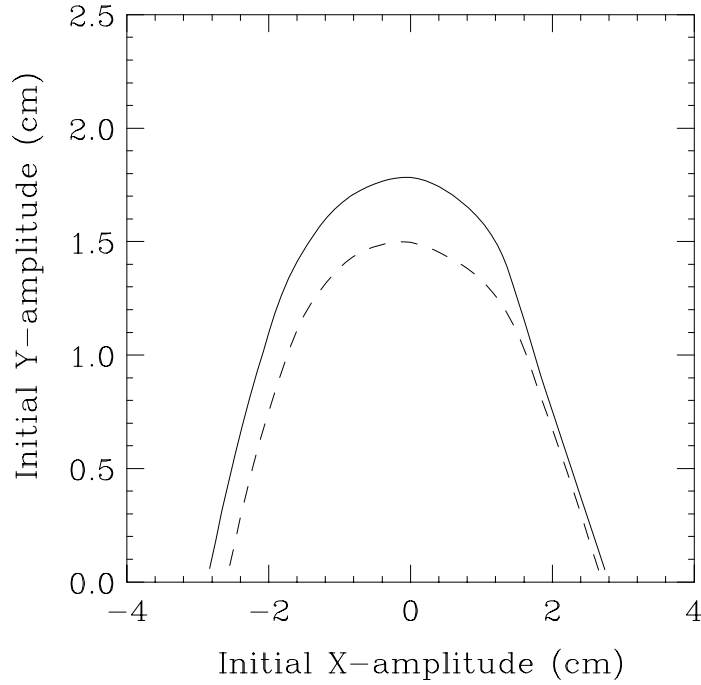


FIGURE 4. Error Free Dynamic Aperture for on-Momentum (solid) and $\delta = 3\%$ off-Momentum (dash) Particles.

is shown in Fig. 4. The axes refer to the initial particle amplitude at the injection point, and the two curves show the boundary for the stable motion for on and off-momentum particles. Clearly, the off-momentum aperture is very robust against momentum errors and provides favorable conditions for a long Touschek lifetime.

EFFECT OF MACHINE ERRORS

Magnetic field and alignment errors introduce optics perturbations and enhance effects of resonances that limit dynamic aperture. For conservative results, we included several different classes of magnet errors in the tracking studies including random main field errors, random and systematic multipole errors, and random alignment errors. Since the skew quadrupoles combined with sextupoles in SPEAR 3 generate a skew octupole field, and the large orbit variation in the 10.6° bend samples high order multipole fields, these effects were added to the error set. The effect of positive linear chromaticity, large β function distortions, orbit distortions, large amplitude coupling and insertion devices were studied. Harmonic sextupole correction has been tried to improve the dynamic aperture, and the effect of lattice periodicity was analyzed.

Alignment and Field Errors

The magnet error and alignment specifications used for the SPEAR 3 tracking studies can be met with standard manufacturing techniques. For tracking simulations, the following values for rms errors were used with 2σ truncation to simulate realistic quality control.

Alignment

The alignment rms errors shown in Table 3 can be achieved with survey techniques used in practice and are large enough to yield conservative tracking results. The dipole alignment specification is the same as for quadrupoles since the SPEAR 3 dipoles include a strong quadrupole field.

TABLE 3. Alignment rms Errors Used in Tracking Studies.

Element	Δx (μm)	Δy (μm)	Roll (μrad)
Dipole	200	200	500
Quadrupole	200	200	500
Sextupole	200	200	500

Systematic Multipole Errors

Systematic multipole field errors are field components of higher order than the main field which apply to all magnets with a common core design. In LEGO, the multipole errors are defined in terms of a ratio of the multipole field ΔB_n (normal or skew) to the main magnet field B at radius r , where $n = 1, 2, 3, \dots$ is the multipole order starting with a bend. The normal rms values $\Delta B_n/B$ used for the SPEAR 3 magnets are listed in Table 4. For the quadrupole magnets, only the allowed multipoles were used. For the gradient dipole magnets and sextupoles combined with skew quads, all systematic multipoles were used with the largest values shown in the Table 4.

In the SPEAR 3 design the skew quadrupoles used for coupling correction are combined with sextupoles. Such combined magnets also generate a skew octupole field proportional to the skew quad strength. At radius of 32 mm the magnitude of the octupole field will be 57% of the skew quad field. In the tracking, the skew octupole component was systematically added in proportion to the skew quadrupole field. It is worth to note that typical skew quad strengths required for coupling correction are less than 1% of the main ring quadrupole strengths.

TABLE 4. Systematic rms Multipole Field Errors.

Magnet	$r(mm)$	n	$\Delta B_n/B$
Dipole	30	2	1×10^{-4}
		3-14	5×10^{-4}
Quadrupole	32	6,10,14	5×10^{-4}
Sextupole	32	4	-8.8×10^{-4}
		5	-6.6×10^{-4}
		9	-1.6×10^{-3}
		15	-4.5×10^{-4}

Random Field Errors

Differences in magnetic core length will give rise to $\sim 10^{-3}$ random main field errors. Normal random multipole errors, introduced by magnet assembly imperfections, are listed in Table 5. To achieve conservative tracking results, large values were specified for the random $n=3,6,10,14$ multipoles on the quadrupole magnets.

TABLE 5. Random rms Multipole Field Errors.

Magnet	$r(mm)$	n	$\Delta B_n/B$
Dipole	30	2	1×10^{-4}
Quadrupole	32	3,6,10,14	5×10^{-4}
		4,5,7-9,11-13	1×10^{-4}
Sextupole	32	5	1.5×10^{-3}
		7	4.8×10^{-4}

Coupling Correction

The initial design of coupling correction employed four independent skew quadrupoles placed in non-dispersive regions in the matching cells. The four skew quads were used to uncouple the 4×4 one turn transfer matrix. However, the skew quad placement in the matching cells did not provide enough variation of the sum and difference phase advance ($\mu_x \pm \mu_y$) for efficient orthogonal correction. Depending on the set of random machine errors, this occasionally led to strong skew quadrupoles and reduced dynamic aperture.

Further study showed that it was beneficial for dynamic aperture to use skew quadrupole components on the chromatic sextupoles located in the dispersive regions of the arc cells [3]. This configuration provided a more orthogonal set of the skew quad positions with reduced strengths. The negative effects, such as induced

vertical dispersion, were small compared to improved aperture and robustness of the correction. In total, 24 skew quads were arranged in four families and placed at their optimum phase positions. One other negative effect is that a skew quadrupole in a sextupole magnet gives rise to a systematic skew octupole field. In our tracking study, the effect of this field did not reduce the dynamic aperture. As a future option, the large number of skew quads in the above scheme allows expansion of the number of independent families with correction of the vertical dispersion as well.

Dynamic Aperture with Errors

For a realistic simulation with errors, LEGO first generates and adds the chosen set of errors to the magnets, then iteratively applies correction schemes to minimize the optics perturbation, and finally tracks particles with a variety of initial amplitudes to define the dynamic aperture. The basic set of correction schemes in LEGO includes tune, orbit, linear chromaticity and coupling correction systems. In the SPEAR 3 simulation, the tune was corrected by using two families of doublet quads in the arc cells. The nominal orbit correction routine in LEGO is based on a three corrector bump scheme, but other techniques can be implemented as well. Typically in this study, the linear chromaticity was adjusted to zero with the two families of sextupoles in the arc cells, while the matching cell sextupole strength was kept constant. The coupling correction was done by using the four family skew quad correction scheme described in the previous section. An RF voltage of 3.2 MV was used to generate synchrotron oscillations for off-momentum particles.

The resultant dynamic aperture for 6 random seeds of machine errors for on-momentum and $\delta = 3\%$ off-momentum particles with the described correction schemes is shown in Fig. 5. The linear chromaticity was set to zero in this case. Compared to Fig. 4 (no errors), the dynamic aperture has reduced by about 20-30%. Since insertion devices were not included in this calculation, this reduction is solely due to the machine errors and quality of correction procedures described above. As in the case of the error free lattice, the off-momentum aperture for machine with errors is comfortably large. As the Fig. 5 shows, the horizontal dynamic aperture is in the range of 18 to 20 mm for all particles within $\delta = \pm 3\%$ momentum range. This provides favorable conditions for a long Touschek lifetime and sufficient room for horizontal injection oscillations.

Positive Linear Chromaticity

Though most of this study was done with the linear chromaticity corrected to zero, in real machines the value of $\xi = \Delta\nu/\delta$ is typically set slightly positive, up to several units, to avoid effects such as head-tail instability. The main impact of the positive chromaticity on the dynamic aperture is from the increased tune spread in the beam. Due to synchrotron oscillations the particles with large momentum errors would sample a larger area on the tune plane and might cross more harmful betatron

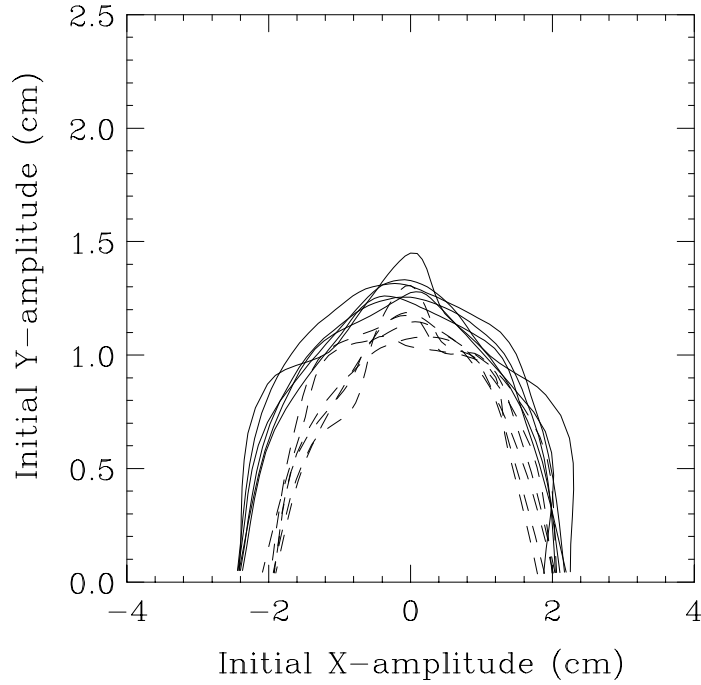


FIGURE 5. Dynamic Aperture for 6 Seeds of SPEAR 3 Machine Errors for $\delta = 0$ (solid) and 3% (dash) Momentum Oscillations.

resonances. As a result, the momentum aperture can be significantly reduced if the linear chromaticity is too large. The effect on the on-momentum aperture is typically smaller due only to the increased strength of sextupole correctors.

Fig. 6,7 show dependence of linear chromaticity on the dynamic aperture for the particles with momentum oscillations of $\delta = 0, 1\%$ and 3% . The tracking included a full random set of machine errors, and the chromaticity was set equal in the x and y planes. Fig. 6,7 show that the particles with momentum errors of $\delta = 1\%$ and 3% lose stability at $\xi \approx 15$ and 6 , respectively. Clearly, this is the effect of a half integer resonance. For the SPEAR 3 working tune ($\nu_x = 14.19$, $\nu_y = 5.23$) the off-momentum particles would likely be lost when momentum dependent tune shift approaches to $\Delta\nu \approx -0.2$. The dynamic aperture for the core beam (low momentum error) is not significantly reduced even for the large positive chromaticity. However, the beam lifetime can be reduced for $\xi > 5$ due to Touschek effect, since the scattered particles with large δ may not survive.

Feed-down Studies

Since the electron beam in a SPEAR 3 dipole magnet follows a 10.6° arc orbit with ± 16.6 mm sagitta, even the particles traveling along the ideal trajectory will sample the full set of high order multipole field. Taking into account the large

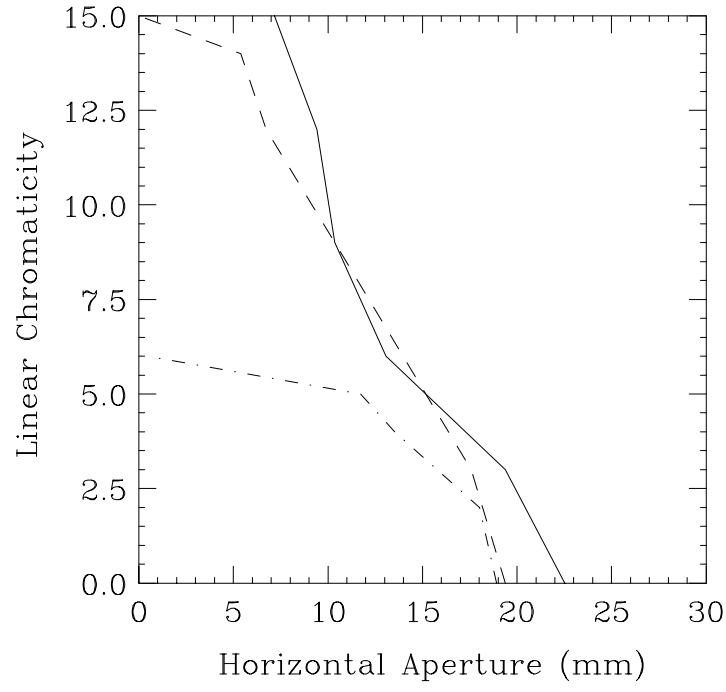


FIGURE 6. Horizontal Dynamic Aperture versus Linear Chromaticity for $\delta = 0$ (solid), 1% (dash) and 3% (dot-dash) Momentum Oscillations.

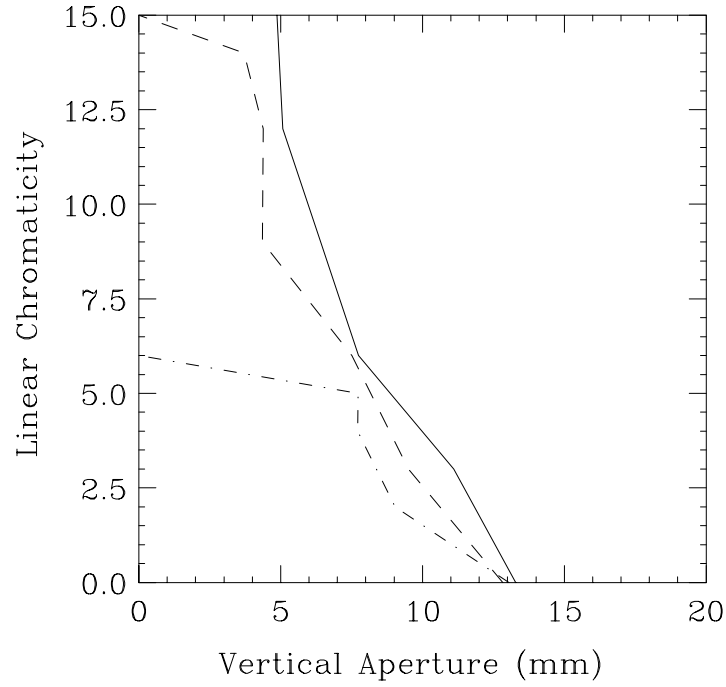


FIGURE 7. Vertical Dynamic Aperture versus Linear Chromaticity for $\delta = 0$ (solid), 1% (dash) and 3% (dot-dash) Momentum Oscillations.

horizontal excursion of the beam orbit, the dipoles are specified to have a full 92 mm good field region [3]. Since normally the Taylor expansion of the multipole field is specified around the central magnetic axis, each individual multipole of this field expanded about the curved beam orbit in a dipole will generate (feed-down) an additional full set of lower order multipoles proportional to the orbit displacement. Both the nominal and feed-down multipoles have to be combined to realistically estimate the effect of dipole errors. Though this feed-down effect might be negligible for large machines, it has to be verified for smaller machines with large sagitta in the dipoles.

In LEGO and most other codes, the multipole field in a dipole would be expanded with respect to the ideal orbit, not the magnetic axis. Therefore, when applying the multipole errors in a dipole, the feed-down terms were included in addition to the nominal set of multipoles specified around the magnetic center. Since the feed-down terms depend on the orbit displacement, we used the following technique to evaluate this effect. The 1.45 m dipole was ‘sliced’ in a reasonably short pieces and the average orbit displacement was calculated for each slice. Based on the orbit displacement in each slice and the nominal set of multipole fields, the systematic feed-down terms were calculated for each slice. The tracking simulation was then done including sliced dipoles with multipole feed-down terms. The results showed no degradation of the dynamic aperture due to this effect.

Since the sliced dipoles would significantly increase the computer time for element-by-element tracking, normally this model was not used in tracking studies. However, the magnitude of the nominal dipole multipole field used in tracking runs without explicit feed-down effects was set conservatively large to produce comparable field around the orbit.

Large Beta Distortion

In the tracking, typical β distortions caused by the SPEAR 3 specification errors after correction were on the order of $\Delta\beta/\beta = \pm(5 - 10)\%$. However, in a real machine it is not unusual to observe much larger modulations since some of the design specifications may not be achieved, especially during commissioning. The β distortions lead to a larger beam size and may increase the effects of high order field and resonances.

To verify the effect of large β modulation on the SPEAR 3 dynamic aperture, the quadrupole errors in two quad families in the matching cells were increased to the level of a few percent to produce $\Delta\beta_x/\beta_x \approx \pm 30\%$ and $\Delta\beta_y/\beta_y \approx \pm 20\%$. The calculated dynamic aperture for 5 seeds of random machine errors is shown in Fig. 8. The average reduction of the aperture due to this β distortion is about 15% compared to Fig. 5. Though this aperture is still adequate to operate the machine, it is clear that such large quadrupole errors have to be identified during initial operation and corrected.

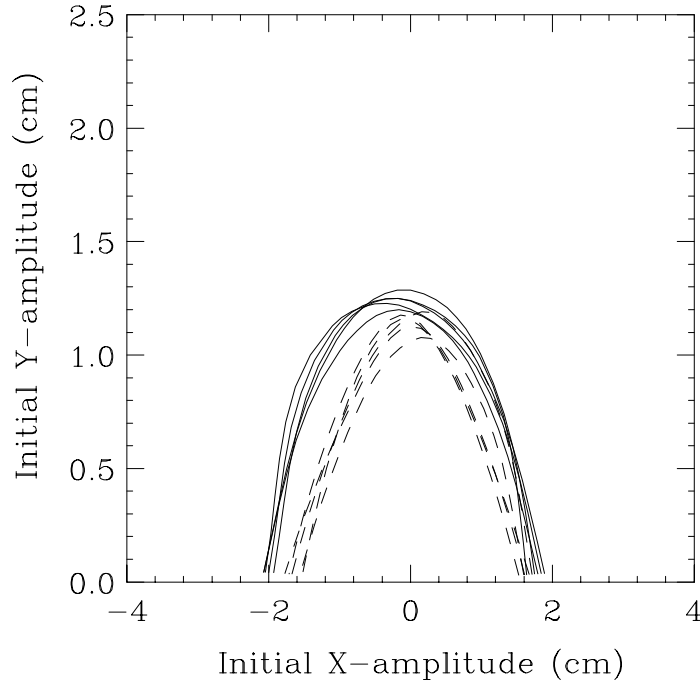


FIGURE 8. Dynamic Aperture with Large β Distortion ($\pm 30\%$ (x), $\pm 20\%$ (y)) for on-Momentum (solid) and $\delta = 3\%$ off-Momentum (dash) Particles for 5 Random Error Seeds.

Orbit Distortion

Large orbit distortions can cause a reduction of dynamic aperture since the electron beam would experience much larger effects from non-linear sextupole and multipole fields. Normally, in this study, the rms orbit was corrected very well, down to the level of $\sim 100\mu m$. To investigate large orbit effects, an additional set of magnet alignment errors was introduced in LEGO just prior to tracking. Using additional uncorrected random alignment errors with $\sigma_{\Delta x} = 80\mu m$, $\sigma_{\Delta y} = 40\mu m$, and $50\mu rad$ roll, rms orbit distortions of up to 3 mm in the horizontal and 1.5 mm in the vertical plane were generated. In each case, the on-energy dynamic aperture was reduced by a maximum of 2 mm in the horizontal and vertical planes for 6 different seeds. The off-energy aperture was reduced similarly compared to the off-energy dynamic aperture without orbit distortions.

For safe machine operation, the peak vertical orbit excursion must be held to < 1 mm and the horizontal excursion should be < 3 mm, so orbit induced reduction of dynamic aperture is negligible. Absolute BPM reading errors are expected to be on the order of a few hundred μm or less. The low sensitivity of the aperture to orbit distortions should simplify initial injection and machine commissioning at low currents.

In a related test, we studied large sextupole misalignment while the orbit was well corrected. The sextupole misalignments generate random quadrupole errors

and result in optics distortion and coupling. To simulate this effect, rms sextupole misalignments of 1 mm were assigned in both planes, and the orbit was corrected down to a few hundred μm level. The 1 mm sextupole displacements generate an order of magnitude larger quadrupole errors compared to the specified field errors in the ring quadrupoles. Of the 6 seeds studied, 5 cases showed > 17 mm horizontal dynamic aperture for on-momentum particles. The vertical aperture was always larger than the ± 6 mm ID chamber size. At $\delta = 3\%$, the horizontal aperture remained above 13 mm. The one 'bad' seed produced 13 mm horizontal dynamic aperture on-momentum and about the same result for 3% off-momentum particles. In practice, with sextupole alignments much better than 1 mm rms, no reduction of dynamic aperture is expected.

Large Amplitude Coupling

In order to monitor the full extent of the dynamic aperture, the SPEAR 3 tracking simulations did not include physical apertures. In practice, however, the vacuum chamber has horizontal and physical apertures that can limit beam lifetime. In the vertical plane, for example, SPEAR has two insertion devices with $y = \pm 6$ mm vacuum chambers that define the vertical acceptance. Although the height of the ID chambers yield acceptable gas scattering lifetime [3], they can limit the Touschek lifetime if strong coupling is present. In the presence of machine errors and strong sextupole fields, for instance, particles with large horizontal amplitudes can reach resonances which couple the horizontal motion into the vertical plane. This effect has been observed in operational machines [15].

To study the effect of large amplitude coupling, we launched particles with variable horizontal and synchrotron oscillation amplitudes and monitored the maximum vertical excursion. Fig. 9 shows the degree of x - y coupling as a function of initial horizontal amplitude for particles with $\delta = 0, 1, 2$, and 3% energy oscillations. Each particle was launched with a small initial vertical amplitude of $100\mu m$ and the peak vertical amplitude was monitored for 1024 turns at the ID location. The plot shows the average value taken over 6 machines with different error seeds. Based on these results, one can conservatively anticipate an effective reduction of horizontal aperture from 20 mm to about 18 mm, and a corresponding reduction in Touschek lifetime from ≈ 135 hrs to ≈ 125 hrs [3]. The 10 mm injection oscillations in the horizontal plane should not be effected by this coupling.

Harmonic Sextupoles

The first non-linear field magnets typically introduced in the optics are sextupoles which are placed in dispersive regions to correct chromaticity. In addition to their chromatic effect, sextupoles generate geometric aberrations such as amplitude dependent tune shift and high order resonances. Even without machine errors these

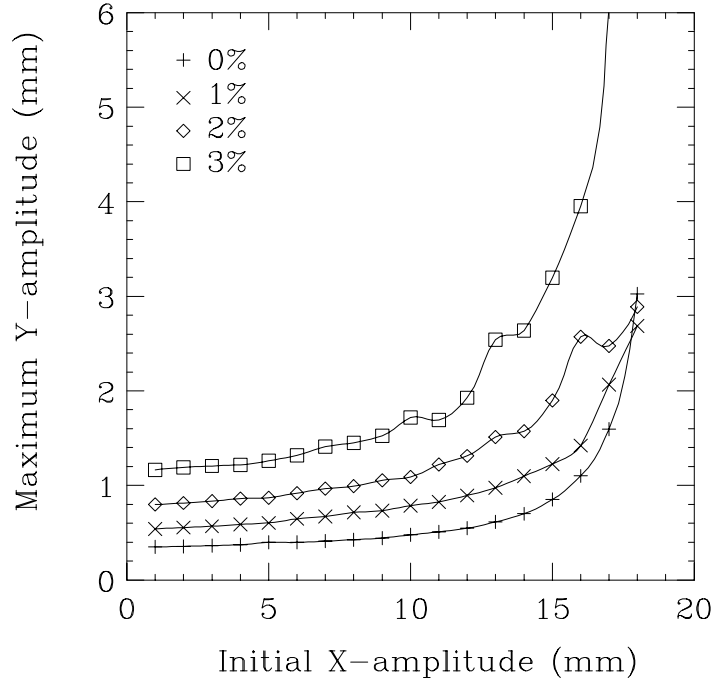


FIGURE 9. Peak Vertical Excursion as a Function of Initial Horizontal Amplitude for $\delta = 0, 1, 2$, and 3% Momentum Oscillation. Results Averaged over 6 Seeds of Machine Errors.

effects may significantly limit dynamic aperture. Therefore, it is important to verify this limitation and use available techniques to minimize a reduction of aperture.

Though the choice of cell phase advance in the SPEAR 3 arcs helps to reduce the sextupole geometric aberrations, they are not completely canceled. To further minimize these effects, additional ‘harmonic’ sextupoles can be used [16]. These sextupoles are usually placed in non-dispersive regions to avoid their chromatic effect, and their strengths are optimized to minimize the total amplitude dependent tune shift. Since this tune shift can be mathematically decomposed into a series of tune harmonic components generated by sextupoles [16], one technique to reduce the tune shift is to minimize the strongest harmonic components.

Obviously, the upper limit for dynamic aperture with sextupoles is the aperture where sextupole aberrations are not present. One way to evaluate this limit is to track on-momentum particles in the lattice with sextupoles turned off. The RF cavities have to be turned off as well to eliminate any chromatic effects. The result of this tracking for SPEAR 3 with 6 random seeds of machine errors is shown in Fig. 10. It follows that if the sextupoles are perfectly compensated the horizontal aperture could be as large as 28 mm or 40% larger compared to Fig. 5. Realistically, this limit may not be achieved since all sextupole aberrations have to be canceled all at once, and even with perfect global compensation the local effects would be present. Effectiveness of a harmonic sextupole system would also depend on the phase advance of a particular optics configuration.

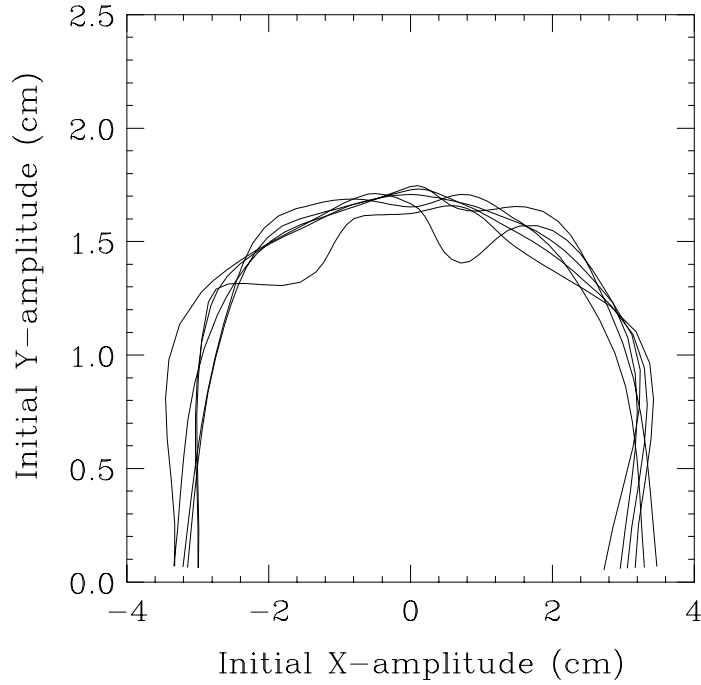


FIGURE 10. On-Momentum Dynamic Aperture without Sextupoles for 6 Random Error Seeds with RF Cavities Turned off.

Based on formulas in [16], we analyzed the magnitude of harmonic components in the amplitude dependent tune shift generated by the chromatic sextupoles. The analysis showed that one of the largest contributions comes from 14th and 18th horizontal tune harmonics. To minimize these contributions we tested a scheme which has two family harmonic sextupoles in each of the 14 arc cells. Because of very limited space in non-dispersive region in the cells, in this test we used thin lens harmonic sextupoles attached to cell quadrupoles in a doublet. The strength of these sextupoles was optimized by scanning and maximizing dynamic aperture. The reduction of tune shift with amplitude due to harmonic sextupoles was verified using HARMON and the results are presented in Table 6, where ϵ is the rms beam emittance. The dynamic aperture calculation with the harmonic sextupoles for error free lattice and machine with errors is shown in Fig. 11,12. Compared to Fig. 4, the improvement of error free dynamic aperture is about 10-15%. However, with machine errors included the improvement of horizontal aperture reduces to a minimum. Taking into account the cost and design complications associated with additional sextupoles as well as marginal aperture improvement, the harmonic sextupole correction was not included in the current design.

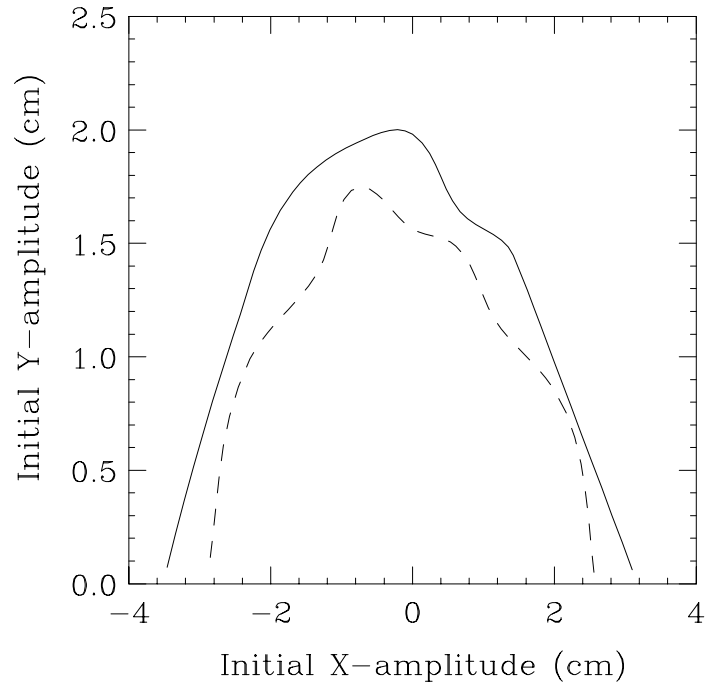


FIGURE 11. Error Free Dynamic Aperture with Two Family Harmonic Sextupoles in Arc Cells for 0 (solid) and 3% (dash) Momentum Oscillations.

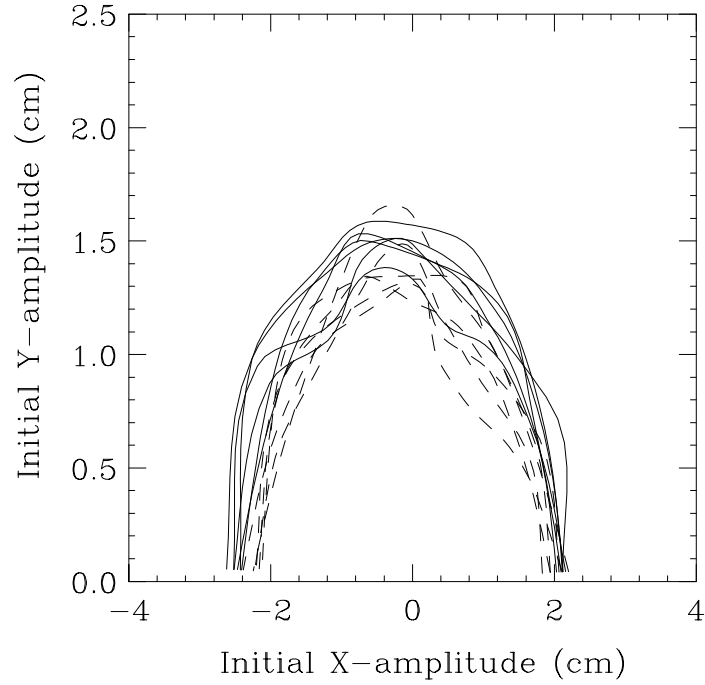


FIGURE 12. Dynamic Aperture with Two Family Harmonic Sextupoles in Arc Cells with SPEAR 3 Machine Errors for 0 (solid) and 3% (dash) Momentum Oscillations.

TABLE 6. Amplitude Dependent Tune Shift for SPEAR 3 with and w/o Two Family Harmonic Sextupoles in Arc Cells.

Amplitude dependent tune shift	$\frac{d\nu_x}{d\epsilon_x}$	$\frac{d\nu_y}{d\epsilon_y}$	$\frac{d\nu_y}{d\epsilon_x}$
w/o harmonic sextupoles	878	1392	643
with harmonic sextupoles	446	866	479

Insertion Devices

At present, seven horizontally deflecting wigglers are planned in the 3 m drifts between arc cells. The typical parameters of these insertion devices are: the peak field B_y up to 2 T, and the total length up to 2.3 m. Though the wiggler magnetic field is highly non-linear, some of its high order effects are locally canceled due to wiggler periodicity. The remaining lowest order perturbations to the beam optics are vertical focusing and amplitude-dependent vertical tune shift due to octupole-like horizontal field. In summary, the wiggler effect on dynamic aperture can arise from perturbation of β functions and tune, high order field effects, reduced periodicity and symmetry of the lattice, as well as wiggler field errors and misalignment.

In SPEAR 3 the wiggler perturbation of β functions and phase advance will be locally corrected using doublet quadrupoles in the cells adjacent to either side of the wiggler [17]. Tracking studies with seven wigglers showed that without high order wiggler effects, the corrected wiggler focusing alone does not reduce dynamic aperture. Without this correction the vertical aperture reduces by about 20%, though it is still well outside the $y = \pm 6$ mm wiggler physical aperture.

The study of wiggler multipole errors was based on recently measured systematic field errors in one of the strongest wigglers (Beamline 11). The measured field data was fit to a set of normal and skew field multipoles up to 12th order and used as wiggler errors in the tracking. Fig. 13 shows that when these systematic multipole errors are included, the dynamic aperture with seven wigglers reduces by about 10%.

The effects of the non-linear wiggler field can further reduce the aperture. Since these intrinsic high order fields are in the horizontal plane, they mostly affect the vertical aperture. Simulation of the first two non-linear terms (octupole and dodecapole-like field) showed rather modest reduction of vertical aperture from 11 mm to 9 mm with the above systematic multipole errors included [17]. This aperture is still well outside the wiggler physical aperture.

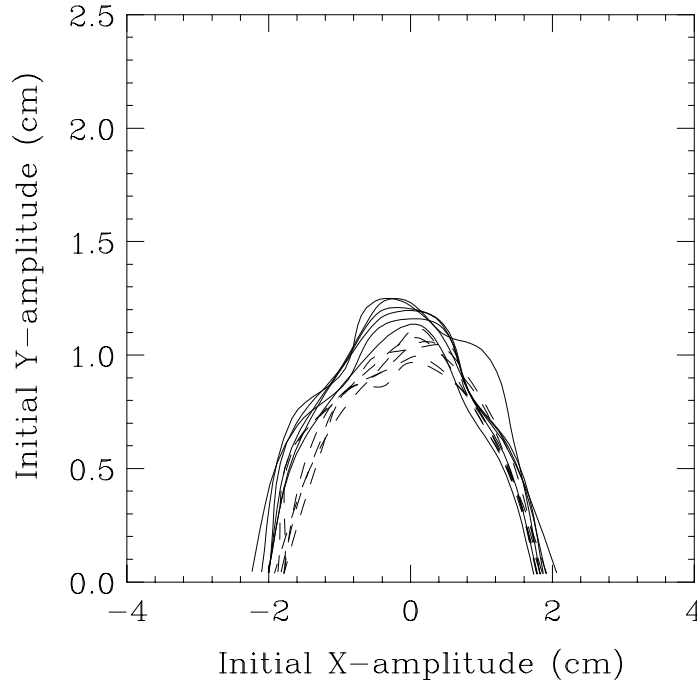


FIGURE 13. Dynamic Aperture with 7 Wigglers, Corrected Wiggler Focusing, Systematic Wiggler Multipole Errors and 6 Seeds of Random Machine Errors for 0 (solid) and 3% (dash) Momentum Oscillations. The Intrinsic High Order Wiggler Fields not Included.

Effect of Lattice Periodicity

The advantage of a high periodicity lattice is that the number of resonances excited by systematic multipole field errors is reduced in proportion to the number of periods. In SPEAR 3 the matching cells break the 14 periodic arc cells and reduce the machine periodicity to 2. To verify the effect of periodicity on dynamic aperture, we tracked particles in a lattice which had only 14 identical arc cells and compared the results to the aperture of the full lattice. The results for on-momentum particles are shown in Fig. 14. The two solid curves correspond to aperture without machine errors, and the dash curves define the aperture for 6 seeds of machine errors. In both cases the pure 14 periodic cells provide a larger dynamic aperture compared to the nominal lattice with matching cells. Since without errors the only non-linear fields are from sextupoles, one can conclude that cancellation of systematic sextupole aberrations is much better in a more periodic lattice. Another observation is that either breaking periodicity or including random machine errors will reduce the SPEAR 3 aperture to about similar size.

Due to geometric constraints in the existing SPEAR tunnel it was unavoidable to break the periodicity of arc cells. To keep the effective periodicity of 14, one of the earliest proposals suggested a *I*-transformation for the matching cell lattice between the arcs [12]. However, the matching cells also contribute a significant amount of

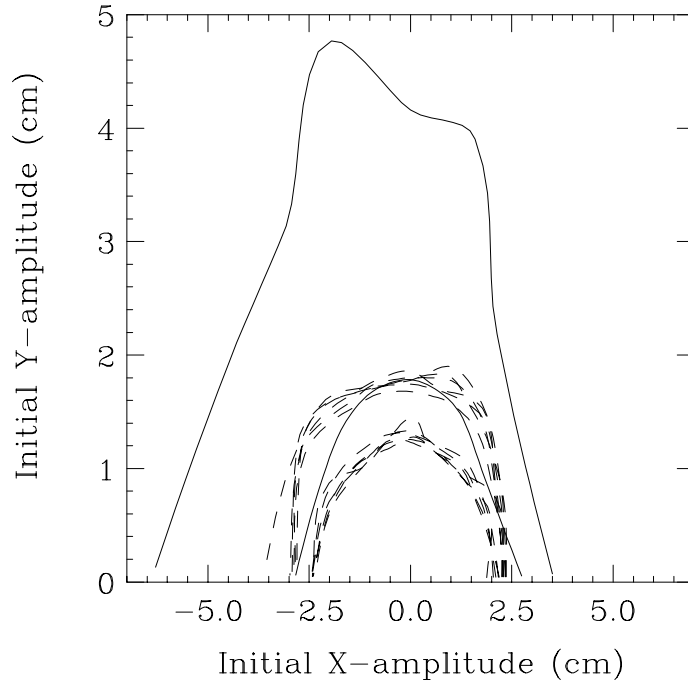


FIGURE 14. On-Momentum Dynamic Aperture: 1) 14 Cells w/o Errors (bigger solid curve), 2) Full Lattice w/o Errors (smaller solid), 3) 14 Cells with Errors (bigger dash), 4) Full Lattice with Errors (smaller dash).

chromaticity, and it was found that relaxing the matching cell optics improved the off-momentum dynamic aperture without compromising the on-momentum aperture.

CONCLUSIONS

A long Touschek lifetime and adequate injection conditions were the primary motivations to maximize the SPEAR 3 dynamic aperture. The described optimization included linear optics, working tune, chromaticity and coupling correction, and compensation of wiggler focusing. Other potential improvements were analyzed as well. The effects of large momentum oscillations, realistic machine errors, insertion devices and larger optics perturbations on dynamic aperture were verified. The results consistently showed that the dynamic aperture for up to $\delta = 3\%$ momentum errors is not significantly affected by these effects. The presented analysis shows that SPEAR 3 dynamic aperture will provide adequate injection conditions and result in > 100 hrs of Touschek beam lifetime.

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